Operation on fuzzy sets

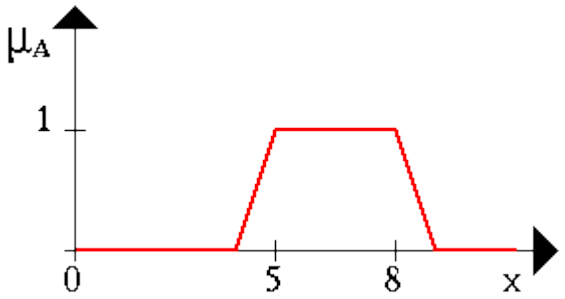
إعداد :

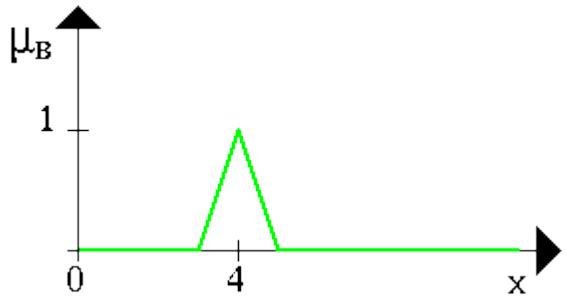
م . يوسف دعبول

Table 1.1 Properties of Classical Set Operations

Involutive law	$\overline{\overline{A}} = A$
Commutative law	$A \cup B = B \cup A$
	$A \cap B = B \cap A$
Associative law	$(A \cup B) \cup C = A \cup (B \cup C)$
	$(A \cap B) \cap C = A \cap (B \cap C)$
Distributive law	$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
	$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
	$A \cup A = A$
	$A \cap A = A$
	$A \cup (A \cap B) = A$
	$A \cap (A \cup B) = A$
	$A \cup (\overline{A} \cap B) = A \cup B$
	$A \cap (\overline{A} \cup B) = A \cap B$
	$A \cup \mathbf{S} = \mathbf{S}$
	$A \cap \varnothing = \varnothing$
	$A \cup \varnothing = A$
	$A \cap \mathbf{S} = A$
	$A \cap \overline{A} = \varnothing$
	$A \cup \overline{A} = \mathbf{S}$
DeMorgan's law	$\overline{A \cap B} = \overline{A} \cup \overline{B}$
	$\overline{A \cup B} = \overline{A} \cap \overline{B}$

Operation on fuzzy sets





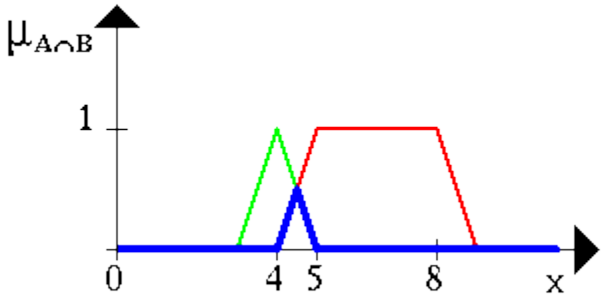
Union and Intersection

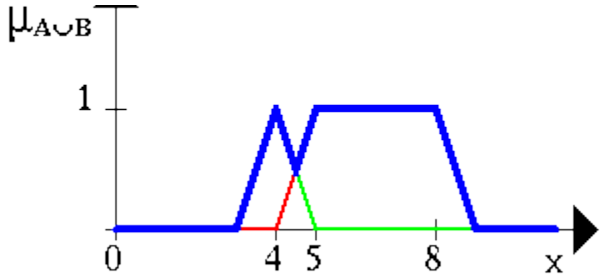
Union:

$$\mu_{\underset{\sim}{A} \cup \underset{\sim}{B}}(x) = \mu_{\underset{\sim}{A}}(x) \mathrm{V} \mu_{\underset{\sim}{B}}(x).$$

Intersection:

$$\mu_{\underset{\sim}{A} \cap \underset{\sim}{B}}(x) = \mu_{\underset{\sim}{A}}(x) \Lambda \mu_{\underset{\sim}{B}}(x).$$

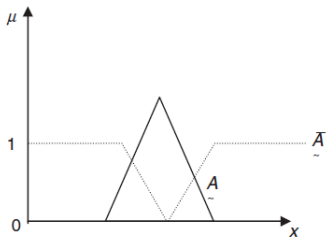




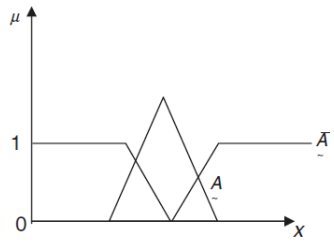
Complement

$$\mu_{\underset{\sim}{\overline{A}}}(x) = 1 - \mu_{\underset{\sim}{A}}(x).$$

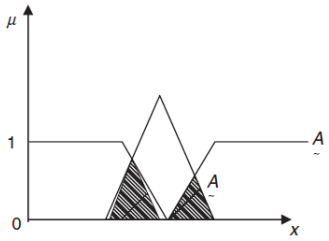
Excluded middle law for fuzzy set



Fuzzy set $\underset{\sim}{A}$ and its complement

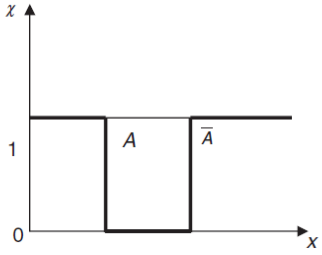


Fuzzy $\underset{\sim}{A} \cup \underset{\sim}{A} \neq x$ (law of excluded middle)

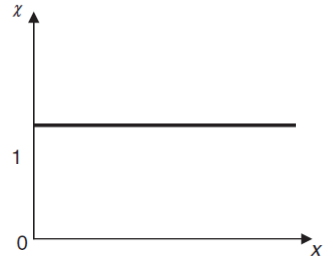


Fuzzy $A \cup \overline{A} \neq \phi$ (law of contradiction)

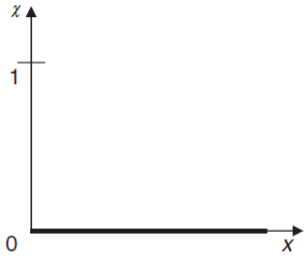
Excluded middle law for classical sets



Crisp set A and its complement



Crisp $A \cup \overline{A} = X$ (law of exclusive middle)



Crisp $A \cap \overline{A} = \phi$ (law of contradiction)

Examples

Consider two fuzzy sets $\underset{\sim}{A}$ and $\underset{\sim}{B}$ find Complement, Union, Intersection, Difference, and De Morgan's law.

$$\underset{\sim}{A} = \left\{ \frac{1}{2} + \frac{0.5}{3} + \frac{0.6}{4} + \frac{0.2}{5} + \frac{0.6}{6} \right\},$$
$$\underset{\sim}{B} = \left\{ \frac{0.5}{2} + \frac{0.8}{3} + \frac{0.4}{4} + \frac{0.7}{5} + \frac{0.3}{6} \right\}.$$

Complement

$$\underset{\sim}{\bar{A}} = \left\{ \frac{0}{2} + \frac{0.5}{3} + \frac{0.4}{4} + \frac{0.8}{5} + \frac{0.4}{6} \right\},$$

$$\underset{\sim}{\bar{B}} = \left\{ \frac{0.5}{2} + \frac{0.2}{3} + \frac{0.6}{4} + \frac{0.3}{5} + \frac{0.7}{6} \right\}.$$

Union

$$\underset{\sim}{A} \cup \underset{\sim}{B} = \left\{ \frac{1}{2} + \frac{0.8}{3} + \frac{0.6}{4} + \frac{0.7}{5} + \frac{0.6}{6} \right\}.$$

$$\underset{\sim}{A} \cap \underset{\sim}{B} = \left\{ \frac{0.5}{2} + \frac{0.5}{3} + \frac{0.4}{4} + \frac{0.2}{5} + \frac{0.3}{6} \right\}.$$

De Morgan's Laws

$$\overline{\underset{\sim}{A} \cup \underset{\sim}{B}} = \overline{\underset{\sim}{A}} \cap \overline{\underset{\sim}{B}} = \left\{ \frac{0}{2} + \frac{0.2}{3} + \frac{0.4}{4} + \frac{0.3}{5} + \frac{0.4}{6} \right\},$$

$$\overline{\underset{\sim}{A} \cap \underset{\sim}{B}} = \overline{\underset{\sim}{A}} \cup \overline{\underset{\sim}{B}} = \left\{ \frac{0.5}{2} + \frac{0.5}{3} + \frac{0.6}{4} + \frac{0.8}{5} + \frac{0.7}{6} \right\}.$$

Using MATLAB

Calculate, $A \cup B$, $A \cap B$, $\bar{A}$, $\bar{B}$ by a Matlab program.

homework

Consider the following fuzzy sets

$$A = \left\{ \frac{0.8}{10} + \frac{0.3}{15} + \frac{0.6}{20} + \frac{0.2}{25} \right\},$$

$$B = \left\{ \frac{0.4}{10} + \frac{0.2}{15} + \frac{0.9}{20} + \frac{0.1}{25} \right\}.$$

Calculate the Demorgan's law $\overline{A \cup B} = \bar{A} \cap \bar{B}$, and $\overline{A \cap B} = \bar{A} \cup \bar{B}$ using a matlab program.

subsethood

$$S(A, B) = \frac{|A \cap B|}{|A|}$$

$$0 \leq S(A, B) \leq 1.$$

Calculate the degrees of subsethood $S(C, D)$ and $S(D, C)$ for the fuzzy sets

$C(x) = \frac{x}{x+1}$ for $x \in \{0, 1 \ldots, 10\} = X$;
$D(x) = 1 - x/10$ for $x \in \{0, 1, \ldots, 10\} = X$